

ON CONNECTIVITY OF THE CLEAN GRAPHS OF COMMUTATIVE RINGS

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Let R be a ring with unity. The clean graph $\text{Cl}(R)$ of a ring R is the simple undirected graph whose vertices are of the form (e, u) , where e is an idempotent element and u is a unit of the ring R , and two vertices $(e, u), (f, v)$ of $\text{Cl}(R)$ are adjacent if and only if $ef = fe = 0$ or $uv = vu = 1$. In this paper, we discuss the connectivity of clean graphs. After ascertaining the minimum degree and corresponding vertices, we determine the vertex connectivity of the clean graph of an arbitrary ring. We also determine all the minimum size cut-sets of the clean graphs. Moreover, we provide some necessary conditions for an arbitrary graph to be minimally connected. Consequently, we characterize the rings whose clean graphs are minimally (edge) connected. Finally, we prove that the clean graph of a clean ring R is minimally (edge) connected if and only if R is a Boolean ring.

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